Deterministic Three-Dimensional Analysis of Long-Range Sound Propagation Through Internal-Wave Fields

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Abstract—A Munk profile and a set of propagating internal-wave modes are used to construct a three-dimensional time-varying ocean sound-speed model. Three-dimensional ray tracing is employed to simulate long-range sound propagation of a broadband acoustic signal. Methods are developed to convert three-dimensional ray-tracing results to acoustic time-domain amplitude and phase measurements. The ocean sound-speed model is defined deterministically, and the model acoustic receptions are analyzed deterministically. A single internal-wave mode that is "spatially synchronized" to an arrival can coherently focus and defocus the acoustic energy. These internal waves can cause an arrival's amplitude fluctuation to mimic Rayleigh fading; however, the time-domain phase is stable, in contradiction to the classical Rayleigh fading environment where the received phase is uniformly distributed. For example, the received power attributed to an early arrival propagated over a 750-km range can fluctuate over 40 dB, while the time-domain phase remains within a quarter of a 75-Hz cycle. The characteristics of the time-domain phase are important for establishing coherent integration times at the receiver.

I. INTRODUCTION

SINCE the mid 1970's, there have been continual advances in the understanding of deep-ocean internal-wave fields and acoustic propagation through them [1]–[7]. Much of the early success was based on a path-integral technique [1]. This technique exploits the geometrical-optics approximation for propagation in the absence of internal waves and integrates the internal-wave effects along this path to estimate statistics on received acoustic observables. Several experiments in the 1970's provided theoreticians measured data on acoustic fluctuation with which to compare their predictions [1]. The observations of these experiments were the received acoustic phase and intensity spectra. These measured spectra agreed with theoretical predictions from the path-integral theory, giving support to their conjecture that internal waves significantly impact long-range, deep-ocean acoustic propagation [3].

Users of the path-integral technique understand the limits of its applicability. Recent long-range, low-frequency ocean acoustic-propagation experiments have gathered precise data that cannot be qualitatively characterized using path-integral techniques [6]. For this reason, numerical simulations have been developed. A successful example of numerical-simulation predictions matching measured acoustic receptions from a long-range propagation experiment is described in [7]. In this work, a two-dimensional (2-D) (range and depth) ocean sound-speed model was constructed, representing the deep-ocean waveguide (deterministically) and the internal-wave field (stochastically) present during the experiment. The internal-wave field model was composed of hundreds of internal waves and was in agreement with the Garrett–Munk spectrum. Parabolic equation (PE) numerical computations were used to simulate acoustic propagation. The experimental measurements showed unexpected acoustic fluctuations; the simulation results also showed these fluctuations, yielding strong evidence that can be attributed to the internal-wave field. In summary, previous and continuing successful work in internal-wave ocean modeling is based on characterizing the statistical properties of the received acoustic signal based on a random-field interpretation of the internal-wave sound speed perturbations and is carried out using a 2-D ocean model.

The research reported in this paper uses a different approach. Numerical simulation is used to study acoustic propagation through internal-wave fields; the approach uses three-dimensional (3-D) modeling and deterministically models both the deep-ocean waveguide and the internal-wave field. The goal is to understand how specific internal waves impact the received acoustic signal, as opposed to understanding how a stochastic parameterization of the internal-wave field affects the statistics of the received acoustic observables. For long-range propagation numerical simulations, the amount of computing power available is a major factor when determining the detail of the ocean model. Since computing power will steadily increase through the years and ocean processes are rather slow-moving (time scales of hours), there is interest in how a given internal-wave field, which could be thought of as a single realization of a random field, deterministically affects acoustic propagation in hopes that future models can estimate and track large-scale ocean processes. This would not only yield valuable oceanographic information, but also may allow signal processors to estimate and remove the effects of these processes on the received acoustic arrival, allowing for...
longer coherent integration times and improved ocean-process estimates.

The last note of this section is a word of caution. Ray tracing (infinite-frequency) computations were used to trace the path of the acoustic propagation, but finite-frequency calculations were employed to calculate the time-domain amplitude and phase of the received acoustic signal. Several internal waves were modeled as opposed to thousands of internal waves as researchers expect exist in the deep ocean. The modeled constituent internal-wave amplitudes were constant over near megameter ranges. Only the final ray-tracing results were studied, and the dynamic focusing and focusing of the wavefront as the wave propagates was not studied, and whether such propagation is physically possible for low-bandwidth acoustic signals was not addressed.

Low-frequency sound transmitted underwater can propagate through the deep ocean thousands of kilometers without catastrophic attenuation. The sound takes multiple paths to the receiver, creating many arrivals. By transmitting a broadband waveform, many of the early, individual arrivals can be resolved. By tracking the received time-domain phase of an individual early arrival, slight changes in transmitter-to-receiver acoustic travel time can be determined. This information can be related to a change in average ocean temperature. The average power of an individual arrival is low. Long integration times are required to obtain sufficient signal energy to reliably estimate the received phase. If the time-domain phase is not stable, coherent integration times are short, and sufficient signal energy cannot be accumulated.

Internal waves are the physical ocean process believed to have the most significant impact on the received acoustic time-domain phase for time scales under 12 h. From experimental results, the amplitude of individual arrivals is reported to fluctuate. Histograms of amplitude values taken over long time periods show a shape that is consistent with Rayleigh fading. A traditional description of Rayleigh fading is that a single arrival is composed of many "micromultipath" arrivals, each taking a slightly different route from transmitter to receiver, and the Rayleigh amplitude is the result of constructive and destructive interference of the multiple arrivals. In this case, the time-domain phase will be unstable.

We conjecture that internal waves can cause the early high-angle arrivals' amplitudes to fluctuate while each arrival maintains stable time-domain phase (precise traveltime). Techau demonstrated that oscillatory perturbations to a range-invariant sound-speed profile can cause the amplitude of early arrivals to change without significantly changing the arrival travel time (phase) [8]. Ray-tracing was used in a 2-D ray-based acoustic propagation, because the internal waves are not traveling parallel to the transmitter-receiver plane. The geometrical-optics approximation is employed so that the wave equation is solved via ray tracing, and the ocean channel is completely described by defining the sound-speed field between the transmitter and the receiver. Timefronts are used to measure the ocean-phase stability, and it becomes clear that timefronts are a valuable characteristic for analysis of sound propagation through a fluctuating ocean [10]. For ray-based acoustic modeling, we show that a traditional 2-D (range and depth) model yields significantly different deterministic results than the more general 3-D model. This implies that transverse acoustic refraction induced by internal waves is significant when modeling deterministic acoustic amplitude fluctuations. PE numerical simulations account for internal-wave diffraction effects as well as refraction. However, with present computing power, these methods must be used with 2-D ocean models. Researchers are faced with a challenging trade-off between using 3-D ray tracing, which includes transverse refraction but neglects diffraction, and 2-D PE methods which do not model transverse effects. Comparing results from these approaches and taking the best of each may lead to the understanding necessary to develop better methods.

By employing the deterministic approach to internal-wave modeling, early evidence is obtained toward identifying the spatially synchronized internal waves from the observed acoustic arrival. Also, we demonstrate the feasibility of modeling internal waves as the ocean process causing the received acoustic amplitude distribution to be Rayleigh; however, the amplitude fluctuation is the result of coherent focusing and defocusing of the received sound and not the result of micromultipath (incoherent) interference. This means the time-domain phase of a received arrival can be stable even though the received amplitude appears to be Rayleigh-distributed.

The remainder of the paper is organized as follows. In Section II, the distinction between "physical ocean models" and "computational ocean models" is discussed, and the models used in this paper are defined. In Section III, spatially synchronized internal waves are defined, and their impact on acoustic propagation is examined in detail using 2-D computational ocean models. In Section IV, spatially synchronized internal waves are investigated using 3-D computational ocean models, and the main results of the paper are presented. Section V concludes the paper with a discussion on 2-D and 3-D computational ocean models.
II. PHYSICAL AND COMPUTATIONAL OCEAN MODELS

Many long-range sound experiments have shown that seasonal variations, internal waves, tides, Rosby waves, currents, and mesoscale eddies impact acoustic transmission [1]. By the mid 1970’s oceanographers had identified internal waves as the most important source of variability on time scales less than 12 h [2]. The sound speed model we use is a combination of a range-invariant Munk sound-speed profile and internal waves. These two contributions to the sound-speed field will be described independently, then combined in a sensible way according to [1].

A. Munk Profile

The Munk profile is a popular academic sound-speed profile for midlatitude deep-ocean regions. The speed of sound is a function of temperature and pressure. Temperature changes dominate the shape of the sound-speed profile near the ocean surface, and changes in pressure dominate the shape of the sound-speed profile below the depth of sound-speed minimum. Propagating sound is refracted toward the depth of minimum sound speed, the sound-channel axis, and thus, the deep ocean acts as a waveguide.

The Munk profile, \( c_{\text{Munk}}(z) \), describes the sound speed as it varies with depth and is used to describe the primary features of the deep-ocean channel [11]. The formula for the sound speed as a function of depth is

\[
c_{\text{Munk}}(z) = c_0[1 + \epsilon(e^{-\eta} + \eta - 1)]
\]

where \( c_0 \) is the minimum sound speed, \( \eta = 2(z - z_0)/B \), \( z_0 \) is the depth of the minimum sound speed, \( \epsilon \) is a dimensionless parameter controlling the overall sound-speed variation, and \( B \) is the buoyancy-decay parameter. The following parameter values are taken to define \( c_{\text{Munk}}(z) \): \( z_0 = 1200 \, \text{m}, \, c_0 = 1480 \, \text{m/s}, \, B = 1040 \, \text{m}, \, \epsilon = 0.006 \).

B. Internal Waves

Internal waves are slowly moving density waves and are similar to the familiar ocean-surface waves, except they occur internal to the ocean. For a detailed description of internal waves see [12]. The displacement of all internal waves with a given wavenumber, \( k \) in radians/meter, can be described in terms of a series of horizontally propagating modes each of the form, \( W_j(k, z) \cos(k_x x + k_y y - \omega_j(k) t) \). Here \( x, y, z \) form a Cartesian coordinate system. \( W_j(k, z) \) is internal-wave mode \( j \) of wavenumber \( k \), and it describes the relative vertical displacement of an isodensity by the \( j \)th internal wave. The internal-wave angular frequency for each mode is denoted by \( \omega_j(k) \) in radians/second. Without internal waves the relevant acoustic propagation would lie in the \( XZ \) plane and the corresponding horizontal direction is the nominal acoustic direction. The angle the internal-wave propagation direction makes with respect to the nominal acoustic direction is denoted by \( \theta \), so that \( k_x = k \cos \theta \) and \( k_y = k \sin \theta \).

The discrete set of internal waves for a given wavenumber, \( k \), is obtained through the “eigen” solutions of the following differential equation (derived from the equations of motion) [12]

\[
\frac{\partial^2 W_j(k, z)}{\partial z^2} + k^2 \left[ \frac{N^2(z) - \omega_j^2(k)}{\omega_j^2(k) - \omega_0^2} \right] W_j(k, z) = 0.
\]  

The physically reasonable boundary conditions are zero vertical displacement at the surface, \( W_j(k, 0) = 0 \), and at the bottom of the ocean, \( W_j(k, z_0) = 0 \). The modes are ordered such that \( \omega_j(k) < \omega_{j+1}(k) \). The inertial angular frequency, \( \omega_i \), is a function of the latitude and is introduced by including the Coriolis acceleration into the equations of motion. The inertial frequency takes values from 0 cph at the equator to 0.083 cph at the poles. The density gradient is described in terms of the buoyancy frequency, \( N(z) \), the frequency at which an isodensity displaced from its equilibrium position will oscillate. A simple and accepted model of the buoyancy frequency is to assume an exponential decrease with depth, \( z \)

\[
N(z) = N_0 e^{-z/B}
\]

where \( B \) is the buoyancy-decay parameter and \( N_0 \) is the extrapolated surface buoyancy frequency. When an external force displaces water from the equilibrium positions, internal waves result.

From measurements and analysis, Garrett and Munk have determined strengths for normalized internal waves [2]. The modes \( W_j(k, z) \) are normalized according to

\[
\int_0^{z_0} (N^2(z) - \omega_0^2) W_j^2(k, z) \, dz = 1.
\]

For a given wavenumber, \( k \), a majority of the internal-wave energy tends to reside in the lower order modes; however, the higher order modes may have an important affect on the received waveform, thus it cannot be ignored. Using a stochastic ocean model, Colosi et al. found that only internal waves of mode \( j < 10 \) significantly impact early-arrival wavefront statistics [7].

Consider \( M \) propagating internal waves present in the ocean that influence acoustic propagation. The \( m \)th internal wave has vertical shape \( W_{j,m}(k_m, z) \) of the \( j \)th mode with wavenumber \( k_m \) and has frequency \( \omega_{j,m} \), all related by (2). In the model, this internal wave is given an amplitude and phase represented by the complex number \( G_{j,m}(k_m, \theta_m) \), where \( \theta_m \) is the horizontal angle of travel of this internal wave relative to the nominal acoustic direction.

The displacement of an isodensity by the \( m \)th internal wave is

\[
\zeta_m(x, y, z, t) = Re \left[ G_{j,m}(k_m, \theta_m) W_{j,m}^m(k_m, z) \right. \left. e^{i(k_m x \cos \theta_m + k_m y \sin \theta_m - \omega_{j,m}(k)m t)} \right].
\]

The aggregate displacement is the sum of these propagating internal-wave displacements

\[
\zeta(x, y, z, t) = \sum_{m=1}^{M} \zeta_m(x, y, z, t).
\]

\( ^1 \)Electrical engineers would call this the impulse response frequency or pole frequency of a circuit.
Following [1], the change in sound speed induced by the internal waves is related to the displacement of the isodensity by
\[ \delta c(x, y, z, t) = 2.5c_0 N^2(z) \zeta(x, y, z, t). \] (7)

In the current research, the complex magnitudes \( G_{j,m} \) are picked to have reasonable or interesting magnitudes consistent with the Garrett–Munk spectrum; they are not random processes of time or space. The amplitude and propagation direction of the internal waves do not change over the 750-km range and four-hour interval used to study their impact on acoustic propagation. We recognize that this coherence may not be consistent with stochastic internal wave theory; they are part of the assumptions made to initiate this current deterministic analysis.

### C. Physical Ocean Models

After establishing a set of internal waves, the 3-D time-varying sound-speed field is completely specified by
\[ c(x, y, z, t) = c_{\text{Munk}}(z) + \delta c(x, y, z, t). \] (8)

Equation (8) is the physical ocean model used for this study. A physical ocean model is a representation of the sound-speed field based on the understanding of the physical ocean processes under study. In other words, no compromises have been made to accommodate for the particular wave-propagation simulation technique. Physical ocean models are 3-D and usually time-varying. The wave-propagation simulation technique used works on a computational ocean model which is typically a subspace of the physical ocean model. For example, a 2-D computational ocean model suitable for PE numerical computations would be to set \( y = 0 \) and fix \( t = t_0 \) in the physical ocean model of (8), yielding the computational ocean model \( c(x, 0, z, t_0) \). One advantage of 3-D-ray tracing is that the computational ocean model can be equivalent to the physical ocean model.

Table I lists the parameters required to define the physical ocean model. Column three of the table defines the parameters or range of parameters used in the simulations. The parameters in the first four rows of the table are selected for each of the \( M \) internal waves. The internal-wave frequency, \( \omega_{j,m}(t) \), is determined by specifying the wavenumber, \( k_m \), and mode number, \( j \), but is included for completeness.

### D. Ray Tracing

In this research, acoustic wavefronts are propagated using conventional ray-tracing equations [15]. Rays are traced in all three spatial dimensions; the independent variable is either time or range. A fourth-order Runge–Kutta integrator is used with an adaptive step size to yield better than 1 ms accuracy at 750 km (approximately 500 seconds traveltime). Surface- and bottom-reflects rays are not considered.

A large number of rays (60,000 to 150,000) are launched from a point source within a small solid angle to simulate a single down-range arrival sheet. Two angles specify a ray launch. The ray inclination at the source in the vertical plane is denoted by \( \alpha_z \) and launch angle in the transverse direction by \( \alpha_y \). A parallel processor with 32 nodes each running at a 66.7 MHz clock rate is used.

Methods have been developed to calculate the internal-wave induced time-domain amplitude and phase fluctuations on the acoustical arrival from ray-tracing computations [14]. The methods are based on the construction of timefronts and not eigen rays so that the amplitude and phase variations can be efficiently computed across an entire wavefront. Also, the “infinite-amplitude at caustics” problem associated with ray computations is avoided; moreover, this amplitude-computation method underestimates large amplitudes and overestimates low amplitudes. Since our results will show large amplitude fluctuations, this calculation method is conservative.

### III. Spatially Synchronized Internal Waves

The phase-stable amplitude-fluctuation impact a single spatially synchronized internal wave has on an early arrival is demonstrated. The acoustic-coherent (phase-stable) focusing and defocusing (amplitude fluctuation) takes place in both the nominal acoustic plane and the \( YZ \) plane, and we call this vertical and transverse focusing, respectively. The dimensionality of the computational ocean model will be increased step by step from a range-invariant time-invariant 2-D model to a complete 3-D time-varying model. The computational ocean models used are in the form of (8). The Munk-profile parameters are listed in Section II. All simulations investigate a single early arrival at a range of approximately 750 km. The rays with vertical-launch angles between \(-12.0^\circ\) and \(-12.6^\circ\)
form the timefront sheet of interest. These rays all have 14 upper turning points, and their endpoints constitute a single downgoing sheet of the timefront after 504 s of propagation. The source is located at the depth of minimum sound speed for all computer simulations.

A. Vertical Focusing

To simplify the description of the deterministic impact of a single internal wave on acoustic propagation, a nontime varying 2-D computational ocean model is used \( c(x, y, z, t) = c(x, 0, z, t_0) \) from (8). The simplest case is a single internal wave traveling perpendicular to the nominal acoustic direction. When \( \theta = 90^\circ \), \( k_x = k \cos \theta = 0 \), describing a range-invariant model where there is no transverse gradient, and sound travels in the nominal acoustic plane.

The internal-wave induced sound-speed deviation, \( \delta c(z) \), at the acoustic-ray top-turning depth is important since this is where the majority of the internal-wave influence on acoustic propagation takes place [5]. An intuitive feel for a single internal wave’s impact on acoustic propagation is developed by considering the shape of \( \delta c \) at the ray’s top-turning depth and recalling that rays bend toward sound-speed minimums. Fig. 1 shows the change in sound speed caused by an internal wave with four sound-speed extrema in the vertical \( \left( J' = 4 \right) \) and horizontal wavenumber \( k/2\pi = 2.05 \text{ cyc/km} \). In a range-invariant ocean model, each ray has a unique top-turning depth; therefore, the underlying rays that form the timefront are refracted by the internal-wave gradient, \( \partial(\delta c(z))/\partial z \), evaluated at the respective turning depths. Just as the Munk profile globally focuses rays to form the deep-ocean channel, the internal wave focuses rays locally. The top-turning depths for rays with launch angles of \( \alpha_z = \pm 14.9^\circ, \pm 12.3^\circ, \pm 9.3^\circ, \pm 5.6^\circ \) are identified in Fig. 1; at these depths the internal-wave induced change in sound-speed gradient is zero. The internal wave focuses and defocuses the acoustic amplitude about these rays. For example, consider a timefront composed of rays with launch angles between \(-12.0^\circ\) and \(-12.6^\circ\). The rays with launch angles above \(-12.3^\circ\) will be refracted downward toward the \(-12.3^\circ\) ray; the rays with launch angles below \(-12.3^\circ\) will be refracted upward toward the \(-12.3^\circ\) ray. In both cases this is due to the locally “cupped” shape of the internal wave. The internal wave focuses the rays toward the unperturbed \(-12.3^\circ\) point on the timefront. By similar reasoning, rays with top-turning depths near 400 m are defocused away from the \(-9.3^\circ\) ray. The natural oscillatory shape of internal waves through the thermocline causes the amplitude of the early high-angle arrivals to be focused or defocused along the timefront. This is called “vertical focusing” since the focusing takes place in the nominal acoustic plane.

To quantify the amount of focusing and defocusing, the power relative to a no-internal-wave ocean reception [see 14 for computational methods] is calculated for the \(-12.3^\circ\) arrival at a range of 750 km. The internal-wave induced power fluctuation is plotted in Fig. 2 for various internal-wave strengths \( \delta c(z) \) at 202 m. When the phase of the internal wave is reversed by a half cycle, defocusing results. It is concluded a single internal wave of modest strength can cause a significant acoustic amplitude change.

B. Spatially Synchronized Internal Waves

When the internal-wave wavelength projected in the direction of sound propagation, \( \Lambda_\theta = 2\pi/(k \cos \theta) \), is an integral multiple of the acoustic-path cycle length, the propagating internal wave can appear to be range independent. The impact of the internal wave is reinforced each time the acoustic wave travels through the upper thermocline. In this case, the internal wave is "spatially synchronized" to the acoustic arrival.

When an internal wave is propagating horizontally at an angle other than \( \pm 90^\circ \) to the acoustic propagation, the 2-D computational ocean model is range dependent, \( c(x, 0, z, t) \). When \( \Lambda_\theta \) is large, i.e., thousands of meters, the internal wave
Fig. 2. Internal-wave induced power fluctuation resulting from coherent focusing and defocusing in range-independent model. The arrival has a top-turning depth of 202 m below the ocean surface.

changes slowly with range. This is referred to as the long-wavelength case. But for internal-wave propagation direction angles away from normal and large-\(k\) internal waves, \(\Lambda_c\) can be as low as a few hundred meters. This is referred to as the short-wavelength case. In this case, the rapid changing of the internal-wave induced sound speed along the top-turning portion of the acoustic path yields a canceling effect, and a spatially synchronized internal wave causes little change in the received arrival amplitude or time-domain phase.

In the long-wavelength case, the effect of spatially synchronized internal waves can be significant. In a frozen ocean model (time-invariant sound-speed field), an internal wave is spatially synchronized with an arrival with acoustic-ray path cycle length, \(L_{\text{ray}}(\alpha_s)\), under the condition

\[L_{\text{ray}}(\alpha_s) = n\Lambda_c = \frac{2\pi n}{k \cos \theta} \tag{9}\]

where \(n\) is an integer. However, many seconds will pass between times when the acoustic wave is at its upper turning point, and the internal wave may have moved significantly. By including time, the spatial synchronization condition becomes

\[L_{\text{ray}}(\alpha_s) = \frac{2\pi n}{k \cos \theta - \omega/c} \tag{10}\]

For a given internal wave, the propagation-direction angles required for spatial synchronization are

\[\theta = \cos^{-1} \left( \frac{2\pi n}{kL_{\text{ray}}(\alpha_s) + \omega/c} \right). \tag{11}\]

When the spatially synchronized internal-wave phase maximizes \(\delta e(z)\), and the acoustic-ray path is at its top-turning point, the internal wave will have the greatest focusing effect. Each time the ray reaches its top-turning point, the internal wave causes a maximum reinforcement. If the internal wave is spatially synchronized with an acoustic-ray path that has a top-turning depth where the internal wave’s change-in-sound-speed gradient is zero (with respect to depth), vertical focusing or defocusing (depending on the phase of the internal wave) results.

Long-wavelength internal waves that are not spatially synchronized to a ray cycle length cause little change in the received time-domain amplitude and phase.

There are three ranges of \(\Lambda_c\): large, midrange, and small; only the large and mid-range causes are effective in spatial synchronization. This characterization has fuzzy boundaries, but roughly a large \(\Lambda_c\) is larger than one-fifth of \(L_{\text{ray}}\), while a small \(\Lambda_c\) is less than 0.04 \(L_{\text{ray}}\). Internal-wave wavelengths projected on the direction of acoustic propagation that are between these extremes are said to be “midrange.” When \(\Lambda_c\) is in the midrange, the amplitude focusing can be very intense (over 10 dB received power increase or decrease relative to a reception propagated through an internal-wave free ocean model).

For an example of midrange \(\Lambda_c\) focusing using a 2-D computational ocean model, consider rays uniformly spaced in launch angle between \(\alpha = -12.6^\circ\) and \(-12.0^\circ\). Fig. 3(a) shows the timefront (solid line) after 504 s for an ocean without internal waves. This is the high-angle early arrival that will be studied throughout the paper. The internal waves used in the computer simulations are listed in Table II. IW1 is used and set to a propagation-direction angle of \(\theta = 86.0^\circ\) to meet the midrange spatial synchronization condition.

The ray endpoints (circles) with no internal wave present are uniformly spaced along the timefront. The internal waves has significantly vertically focused the ray endpoints (+), yet they lie nearly on top of the same timefront. This means the internal wave focuses the amplitude along the front but does not significantly alter the time-of-arrival, or equivalently the time-domain phase of the waveform. Fig. 3(b) shows the change in arrival power. For this model, a hydrophone at a depth of
Fig. 3. Vertical focusing in forced 2-D range-dependent internal-wave model. (a) Timefront for early arrival without internal-wave perturbation and corresponding ray endpoints (C). Also, the ray endpoints resulting from an ocean model, including a single propagating spatially synchronized internal wave, are marked by (+), and these points fall nearly on the internal-wave free timefront, indicating the internal wave does not induce a significant phase shift on the received acoustic signal. (b) The internal wave induces a 17 dB gain in power at a specific location on the front.

Table I

<table>
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<th>Name</th>
<th>j</th>
<th>$k$ (cyc/km)</th>
<th>$\omega_j$ (cph)</th>
<th>$G$ (magnitude and phase)</th>
<th>$\delta c_{max}$ (m/s)</th>
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1250 m and range near 748 km would enjoy a 17-dB gain due to the presence of the internal wave.

IV. 3-D COMPUTATIONAL OCEAN MODELS

In this section, 3-D time-varying computational ocean models are employed to study acoustic propagation through internal-wave fields.

A. Three- Versus Two-Dimensional Modeling

Internal waves can induce a sound-speed gradient normal to the direction of sound propagation. This will result in refraction of the ray paths out of the nominal acoustic plane. The wavelength of the internal wave in the transverse direction is $2\pi/(k \sin \theta)$. If the ray deviates by $\pi/(k \sin \theta)$ in the transverse direction, the internal-wave sound-speed perturbation has exactly the opposite polarity as the sound-speed perturbation used in the 2-D computational model. For high-wavenumber internal waves, $\pi/(k \sin \theta)$ can be as low as a few hundred meters.

In the next few sections, the internal-wave induced power fluctuation across the 3-D early arriving front is studied. Figs. 4 and 5 look at rectangular portions of a sheet of a timefront to show the effect of internal waves on the received intensity. The situation and description of what will be plotted will be described in detail for Figs. 4 and 5.

The sound speed used is a range-invariant Munk profile with a single internal wave present. The traveltime is 504 s. Each plot is the result of propagating 150000 rays using 300 vertical angles between $-12.6^\circ$ and $-12.0^\circ$ and 500 horizontal angles between $-0.5^\circ$ and $+0.5^\circ$. These 150000 rays terminate on a timefront sheet at a range of about 748 km, extending approximately 2.3 km vertically at an angle of about $12.3^\circ$ off vertical and propagating forward and downward.

Fig. 4 uses gray scale to indicate the fluctuation in received power caused by the presence of a single internal wave; specifically, it indicates the ratio of the received intensity with the internal wave relative to the received intensity without the internal wave. Light colors indicate increased intensity, and dark colors reduced intensity. The midrange gray of 0 dB means the internal wave has caused no amplitude change. The gray scale is shown at the bottom of Fig. 4.

Two different internal waves were used. Both internal waves are spatially synchronized for the $12.3^\circ$ ray. IW1 caused the
upper plots; IW2 caused the lower plots. The internal wave parameters and the reasons for selecting these specific internal waves will be explained while discussing the results.

Two different computational ocean models were used. Without internal waves these yield identical results. The plots to the right used full 3-D computational ocean models and ray tracing. The plots to the left used 500 x 2-D computational ocean models, meaning the ray leaving the source at each of the 500 horizontal angles was forced to stay at that angle by changing the transverse wavenumber to zero. (called N x 2-D in the literature where here N = 500).

IW1 is the same internal wave as used for Fig. 3, where it demonstrated how an internal wave shows vertical focusing using a 2-D computation. The upper left plot shows a fuller picture of vertical focusing using 500 x 2-D computation; it shows the vertical focusing has a sharp depth sensitivity that shifts in the transverse direction with depth. The upper right plot shows the true focusing using 3-D computation. The angling to the right with depth is parallel over the entire sheet; the central focusing seen in the 500 x 2-D results are bordered by broad low-amplitude bands, and narrow high-amplitude bands about 1.5 km either side of center.

IW2 is a first order (j = 1) internal wave moving almost perpendicular to the acoustic propagation. The lower left N x 2-D computation sees almost no internal wave effects. In contrast, the lower right 3-D computation shows parallel vertical banding amid a string-like structure of varying intensification and nulls. This is a phenomenon that will never be seen using a 2-D or N x 2-D computational ocean model, where acoustic rays are forced to remain in the nominal acoustic plane. This "transverse focusing" will be treated in Section IV-B.

This section has demonstrated that 3-D computations reveal intensification patterns that are, at best, only hinted at by the 2-D and N x 2-D computations. There has been no attempt at statistical computations using a stochastic ocean model, and it may well be true that some statistical intensity values based on 2-D computations give insight to the values derived from 3-D computations. This study does not address a comparison of acoustic-phase variations for 2-D and 3-D ocean models.

B. Transverse Focusing

In the previous section, it was demonstrated that the oscillatory shape of an internal wave across depth vertically focuses the acoustic rays. In the transverse direction, the change in sound speed \( \delta c(y) \) varies exactly sinusoidally for all internal waves as seen by (5). Just as the cupped shape of the internal wave in the depth direction vertically focuses the acoustic rays, the sinusoidal shape of \( \delta c(y) \) transversely focuses the acoustic rays. A spatially synchronized internal wave will significantly focus the sound transversely. Transverse focusing is typically a larger contributor to the redistribution of power across the front than vertical focusing.

Four examples of transverse focusing are demonstrated by including various spatially synchronized internal waves in the ocean model. The same early arrival investigated in the previous section is studied. For each example, the fluctuation in received power caused by the presence of the internal wave...
is displayed in Fig. 5. A 3-D computational ocean model is used for all cases.

Consider internal wave IW1. The internal-wave propagation direction is set to $-77.5^\circ$ to allow for spatial synchronization. The received power-fluctuation results are shown in the top left plot of Fig. 5. The presence of the internal wave increases the received power by more than 15 dB along a 100-m-wide canted area on the timefront. Broad 400-m-wide 15-dB fades are set parallel to either side of the focus area. A transverse shift of 100 m in hydrophone placement could change the received power at this time instant by over 30 dB. The internal wave presents little impact to the received power for 1-km shifts away from the focus area. This is where the effective propagation angle no longer meets the spatial-synchronization condition.

In Fig. 5, the top right plot shows results from the same internal wave (IW1) used in the adjacent plot. Also, the internal wave is traveling in the same direction with respect to acoustic propagation. The only change is that 23 min have passed, equal to half of an internal-wave period ($\pi/\omega$). The polarity of the internal-wave induced change in sound speed is reversed, and the internal wave transversely defocuses the rays at the top-turning depth, and the 15-dB increase in received power has been replaced by a 15-dB decrease in received power.

Internal waves with large wavenumbers have the potential to have very short wavelengths in the transverse direction. This can cause very sharp and localized focusing. Low-wavenumber internal waves can cause very broad focusing and defocusing. Consider an ocean model including IW3. The propagation-direction angle is set to $84.7^\circ$ to meet a spatial-synchronization condition. The phase of the internal wave was set to defocus the acoustic rays. The power fluctuation caused by the internal wave across the front is shown in the bottom left plot in Fig. 5. The plot covers 40 km transversely to show the broad defocusing. Internal waves with low wavenumbers have correspondingly low internal-wave frequencies, $\omega$. These broad focusing/defocusing internal waves can induce power gains/fades of several dB across an entire array that may last for hour-long periods. The width of the 10-dB power fade is almost 10-km wide.

C. Multiple Internal-Wave Model

Previously, only a single internal wave was included in the ocean model. Spatially synchronized internal waves locally act on the 3-D timefront to spatially redistribute power. When multiple internal waves are included in an ocean model, the set of internal waves generally act on the front as a composite of individual internal waves. No strong statement about linearity can be made, but the system is very well behaved in this manner. In the bottom right plot of Fig. 5, the ocean model includes the seven internal waves listed in Table II. Only IW1 is spatially synchronized; it is traveling at an angle $\theta = +77.5^\circ$ relative to the nominal acoustic direction. Two points will be made from observing this plot. The acoustic rays are focused as if only IW1 is present. Also, the focusing is canted in the reflected and opposite direction as compared
to the results of the top left plot in Fig. 5 where IW1 is used with $\theta = -77.5^\circ$.

Since the amplitude focusing can essentially be contributed to the spatially synchronized internal wave, and nonspatially synchronized internal waves create weak acoustic-amplitude fluctuations, only the spatially synchronized internal waves within a full GM spectrum need to be modeled to study the gross amplitude fluctuation of an early acoustic arrival. This means a 3-D time-varying computational ocean model can be used that is not exactly equivalent to the 3-D time-varying physical ocean model (GM spectrum of internal waves) but may adequately represent it for certain investigative purposes.

By modeling long-range sound propagation through a single internal wave, we have discovered that spatially synchronized internal waves cause canted areas of power gains and fades across the front. Since an internal wave is synchronized with respect to a specific arrival’s acoustic-cycle length, different arrivals will interrogate the ocean with different cycle lengths. Studying all resolvable arrivals may feasibly yield a picture of the internal waves present in the ocean. To begin in this direction, we observe that the cant of the transverse focusing area is solely a function of the internal-wave propagation-direction angle, $\theta$. For cases when a spatially synchronized internal wave is traveling near normal to the direction of acoustic propagation, the focusing pattern is nearly vertical. This is seen in the bottom left plot of Fig. 7. Other cases can be examined in the other plots.

After propagation through an internal-wave field, the 3-D timefront forms a thin corrugated sheet. Computation of the micromultipath combining loss (MMCL) shows instantaneous power losses less than a fraction of a dB with respect to a 75-Hz signal at a 750-km range. The MMCL is a measure of the power loss resulting from internal-wave induced breaking of the wavefront into several arrivals (called micromultipath) that differ in traveltime (phase) and thus combine to some degree destructively. For this research, internal-wave induced micromultipath is not a significant contributor to the power fluctuations, and these fluctuations are solely a result of coherent focusing and defocusing. The MMCL increases with range and internal-wave magnitudes $|G|$. For the internal-wave magnitudes considered in this paper, the MMCL remains less than 3 dB for ranges up to 2 Mm, while internal-wave focusing and defocusing causes power fluctuations over a 40-dB range. One may say that ray chaos [17] is not significant for the ranges and internal-wave strengths studied.

D. Received Power (and Phase as a Function of Time

In this section, the time-domain phase and received power at a hydrophone(s) as a function of time is analyzed. The same early arrival, studied in the previous sections, is investigated. It will be demonstrated that the time-domain phase can remain relatively stable while the received power fluctuates widely both spatially and temporally. At a single hydrophone, the received amplitude distribution may appear to be Rayleigh-distributed while the time-domain phase is relatively stable.

The ocean model includes the seven internal waves in Table II. IW1 and IW2 are spatially synchronized to the acoustic-arrival propagating at angles $\theta = 77.5^\circ$ and $89.2^\circ$, respectively. The magnitude of the maximum change in sound speed induced by the internal-wave field is less than 0.5 m/s.

1) Single Hydrophone Results: The received time-domain power and phase of an arrival at the range of 750 km, depth of 1200 m, and 0 m transverse shift is computed every 5 min over a four-hour period and shown in Fig. 6.
power fluctuations spread over 30 dB. The time-domain phase oscillates with a peak-to-peak deviation of 0.18 cycles at a 75 Hz center frequency; this would impose no limitation on coherent integration.

The intensity level at a candidate receiver site is considered as proportional to the local ray-endpoint density on the timefront sheet. The intensity histogram is computed for several receiver locations distributed across the timefront over the four-hour period. The histograms from the sample receiver locations are averaged, and the resulting histogram is shown in Fig. 7. The intensity histogram is approximately exponential (linear on a log scale) which is consistent with Rayleigh amplitude fading. There is no underlying stochastic process forcing the amplitude distribution to be Rayleigh; it just happens to be similar. Here we show through deterministic modeling that internal waves can cause the received amplitude to mimic Rayleigh fading while maintaining stable time-domain phase.

2) Vertical and Horizontal Arrays: In Fig. 8, the received power and time-domain phase of an early arrival at three hydrophones at a depth of 1200 m, range of 750 km, and transverse coordinates of −300, −150, and 0 m are plotted. The power fluctuates over 30 dB at each hydrophone. At a given time, the received power at two hydrophones with 150 m separation can differ by over 20 dB. Clearly, for a horizontal array, we cannot expect the arrival to be an equal amplitude plane wave across all hydrophones. The time-domain phase at three hydrophones is very similar. This means a time-delay beamformer will work as designed when assuming a plane-wave reception. Similar results are obtained for hydrophones aligned in a vertical orientation.

V. CONCLUSION

Researchers would like to use a computational ocean model consistent with the current physical understanding of internal waves, a full complement of modes generated from (2) in accordance with the Garrett–Munk spectrum. Unfortunately, modern computer limitations do not allow implementation of a full 3-D time-varying internal-wave model over megameter ranges using the method of (8). To use this approach, compromises must be made. Recently, Colosi et al. used a 2-D computational ocean model and represented the Garrett–Munk spectrum by using hundreds of internal waves; the statistical model results were in agreement with experimental results [7]. In this paper, a full 3-D time-varying computational ocean model was used but the number of internal waves was limited to make the model computationally implementable. Fortuitously, we showed early acoustic arrivals are sensitive to a reduced class of (spatially synchronized) internal waves. In some sense, it is possible that these internal waves may fairly represent the Garrett–Munk spectrum in that we are modeling the few modes (from a much larger set of modes) that significantly impact this specific acoustic reception.

Ray tracing was used because it is an efficient computational method for 3-D range-variant environments, and only the rays which may correspond to the single arrival under study need be traced.

As previously mentioned, it is common to represent the physical ocean model using a 2-D computational ocean model in range and depth. This model is obtained from the 3-D model by setting \( y = 0 \) in (8). However, the actual 3-D model corresponding to this 2-D model is one where all internal waves are propagating in the direction of sound propagation (or the exact opposite direction); thus, \( \theta^m = 0^\circ \) or \( 180^\circ \), so that \( k_y = 0 \) and \( k = \pm k_x \). But when \( y \) is set to zero, the term \( k^m y \sin \theta^m \) is effectively removed from (8), and a new set of effective \( k \) values are generated, namely \( k^m \cos \theta^m \), that have corresponding modes, \( W_j^m(k^m, z) \), and frequencies, \( \omega_j(k) \), that are not consistent with (2).

One way researchers model a 3-D field is to propagate the sound over several 2-D slices of the ocean, called \( N \times 2 \)-D modeling. Since for each slice the sound is constrained to the 2-D slice in which it originates, we have the same physical interpretation as the forced 2-D case. For this reason, even
Fig. 8. Horizontal array. (a) Power as a function of time for hydrophones at range of 0.75 Mm, depth of 1500 m, and transverse coordinates of −300 m, −150 m, and 0 m. (b) Time-domain phase for same three receptions.

considering an infinite number of 2-D slices, this model would not approach that of a 3-D model when internal waves are present.

A foundation is established for analyzing 3-D ray-tracing results for long-range sound propagation through internal wave fields. A 3-D time-varying ocean sound-speed model composed of the Munk profile and internal waves is used. Internal waves that are spatially synchronized to the acoustic arrival can coherently focus and defocus the received power across the front. The fluctuation in power across the arrival can be greater than 40 dB without creating unstable phase. Internal waves that are not spatially synchronized do not significantly change the received arrival power. It is feasible that spatially synchronized internal waves are the mechanism causing the received waveform to Rayleigh fade in amplitude coincident with stable time-domain phase. From a deterministic perspective, it has been demonstrated that 3-D computations reveal intensification patterns on the propagating acoustic front that are, at best, only hinted at by the 2-D or $N \times 2$-D computations.

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REFERENCES


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